Generalized synchronization of spatiotemporal chemical chaos

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We report the synchronization of two spatially extended chemical systems. In one spatial dimension, under appropriate parameter conditions, the model systems exhibit a transition to turbulence via backfiring of pulses. By implementing continuous control to the underlying partial differential equations synchronization is achieved not only for identical systems, but also for systems operating under unequal parameter values exhibiting different dynamical behavior. Using this technique, spatiotemporal chaos (turbulence) can be suppressed, maintained, or even enhanced depending on the dynamical behavior of the drive system. This could possibly be of relevance to biological systems, where in certain situations the emergence of chaos is undesirable while under different circumstances the loss of the chaotic dynamics is undesirable (epileptic seizures). [S1063-651X(97)09808-5]

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Interest in the investigation of complex spatiotemporal behavior observed in extended systems has steadily increased over the past few decades. These systems exhibit not only rich phenomological behavior such as pattern formation, solitons, wave propogation, intermittency, and turbulence, but are also extremely relevant to the understanding of fluid, chemical, and biological systems where both spatial and temporal dependences need to be considered [1–4]. More recently interest in synchronization of chaos has increased because of its possible relevance to secure communications. It started out with efforts to synchronize identical systems [5–9], but lately much emphasis has been placed on synchronization of nonidentical systems [10–12] (generalized synchronization). In this paper both synchronization and generalized synchronization of complex spatiotemporal dynamics (including turbulence) associated with unidirectionally coupled spatially extended systems are reported. The synchronization is made possible using the continuous con-

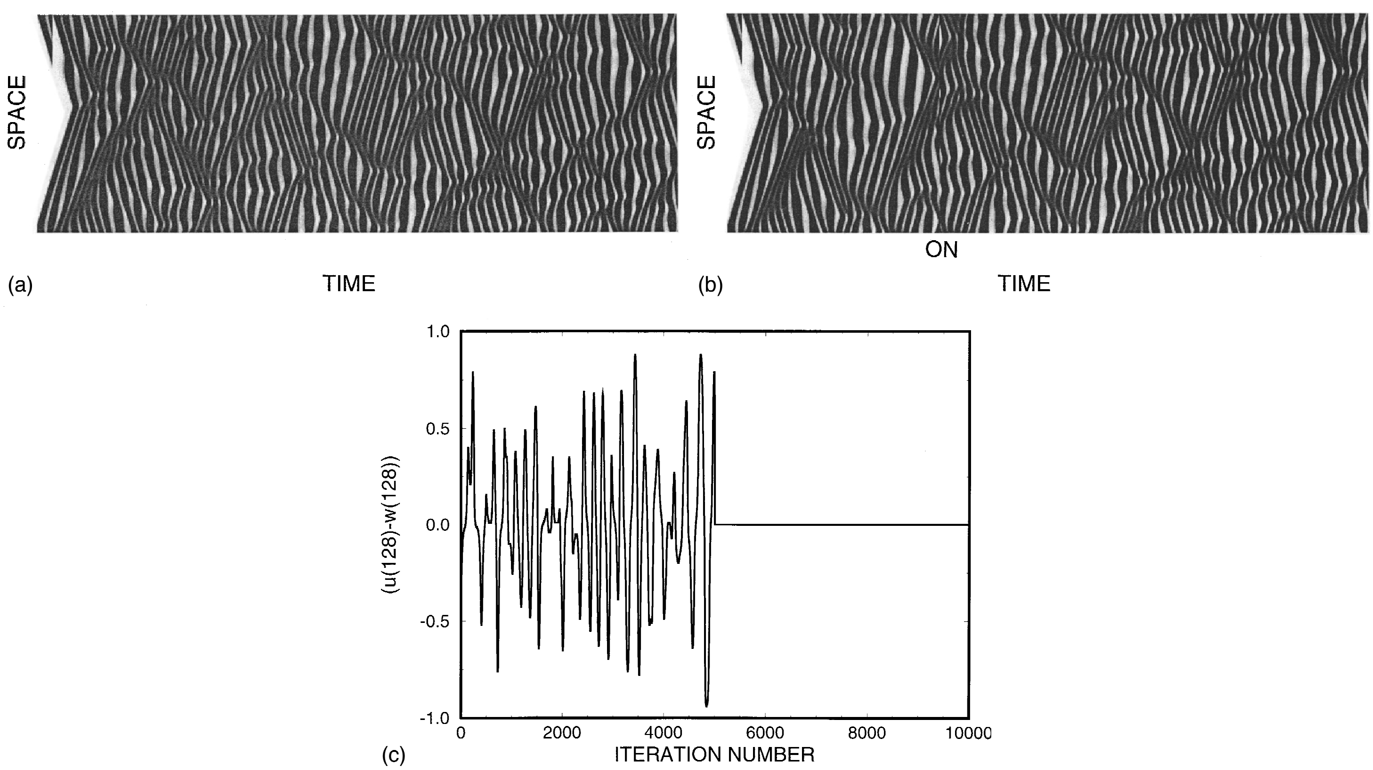

FIG. 1. Space-time plots for identical one-dimensional chemical models [Eqs. (1) and (2)] (initialized at different initial conditions) studied under periodic boundary conditions with the dimensionless system length $L=100$ and the number of grid sites $N=256$. The system parameters are $a=0.84$, $\epsilon=0.39$, and $b=-0.045$. (a) Dynamics of the drive system showing amplitude turbulence. (b) Dynamics of the response system subsequent to initiation of the synchronizing feedback K = 0.07 (indicated by ''ON'' on the plot) converges onto the dynamics of the drive system. (c) Difference in the local dynamics [(u_i-w_i), proportional to the synchronizing feedback] at grid element 128. It indicates clearly the synchronization of the local dynamics subsequent to which the feedback vanishes.

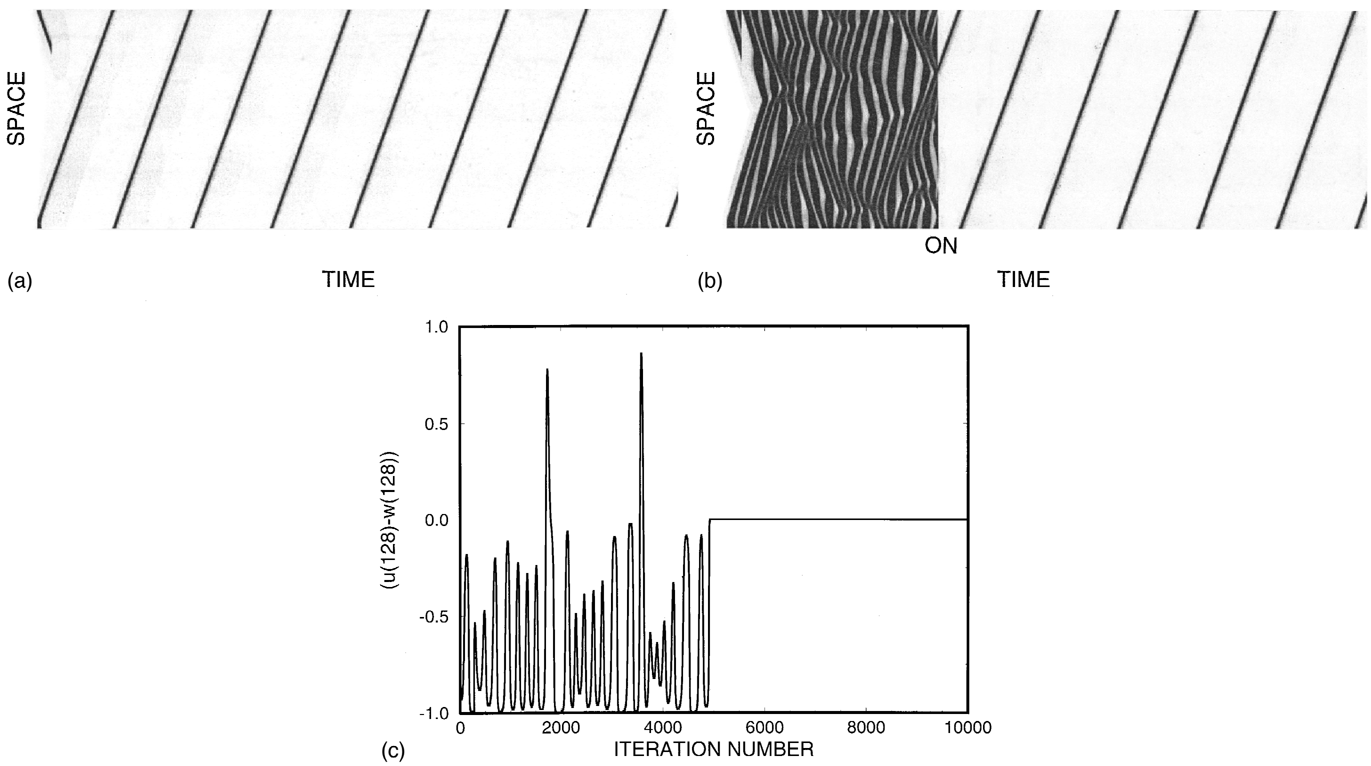

FIG. 2. Space-time plots for nonidentical one-dimensional chemical models [Eq. (1) and (2)] studied under periodic boundary conditions with the dimensionless system length $L=100$ and the number of grid sites $N=256$. (a) Dynamics of the drive system for the system parameters $a=0.84$, $\epsilon=0.09$, and $b=0.07$ exhibiting propogating pulse phenomena. (b) Dynamics of the response system at the system parameters $a=0.84$, $\epsilon=0.39$, and $b=-0.045$ prior to initiation of the synchronizing feedback exhibits turbulence. However, subsequent to switching on the feedack (indicated by ''ON'' on the plot) with $K = 0.1$ the dynamics of the response system converges onto the dynamics of the drive system, namely, a propogating pulse. (c) Difference in the local dynamics [(u_i-w_i), proportional to the synchronizing feedback] at grid point 128. It indicates clearly the synchronization of the local dynamics subsequent to which the feedback vanishes.

trol technique [13–15] requiring knowlege of local dynamics. The paper is organized as follows. First we introduce the numerical model (stemming from catalysis) extended in one spatial dimension exhibiting complex dynamics including turbulence. Then we describe the continuous synchronizing feedback superimposed to the dynamical evolution of the response system. Finally, numerical results showing synchronization and generalized synchronization (systems at different parameter values) of dynamics between the drive and the response systems are presented.

To demonstrate spatiotemporal synchronization, we choose the following model used for description of CO oxidation on a Pt(110) single-crystal surface under UHV conditions [16,17]:

$$\partial_t u = -\frac{-1u(u-1)}{\epsilon}\left(u-\frac{v+b}{a}\right)+\nabla^2 u, \tag{1}$$

$$\partial_t v = f(u)-v, \tag{2}$$

where the activator variable u corresponds to the coverage of the adsorbed CO, while the inhibitor variable v describes a structural change. The function $f(u)$ is of the form

$$u<1/3 \rightarrow f(u)=0,$$

$$1/3 \leqslant u \leqslant 1 \rightarrow f(u)=1-6.75u(u-1)^2, \tag{3}$$

$$u>1 \rightarrow f(u)=1.$$

Under appropriate parameter values in one spatial dimension, the model system exhibits traveling pulse behavior, amplitude turbulence, and phase turbulence. The system size was chosen to be 100 (dimensionless units) and was divided into 256 grid elements for simulation of the model using an explicit integration algorithm with constant time and space steps (100/256) subjected to periodic boundary conditions. The above system drives a replica of itself at the same parameter values (corresponding to synchronization) or at different parameter values (corresponding to generalized sychronization).

To achieve synchronization, we use the difference $K(u_i-w_i)$ [the subscript i corresponds to the grid element $(1 \leqslant i \leqslant 256)$] and superimpose it onto the dynamical evolution of the response system represented by

$$\partial_t w = -\frac{-1w(w-1)}{\epsilon}\left(w-\frac{x+b}{a}\right)-K(u_i-w_i)+\nabla^2 w, \tag{4}$$

$$\partial_t x = f(w)-x, \tag{5}$$

where $f(w)$ has the same functional form as $f(u)$ in Eq. (3). When the control parameter $K=0$, it corresponds to the unsynchronized case, whereas triggering K to nonzero values initiates synchronization via continuous feedback. So, in

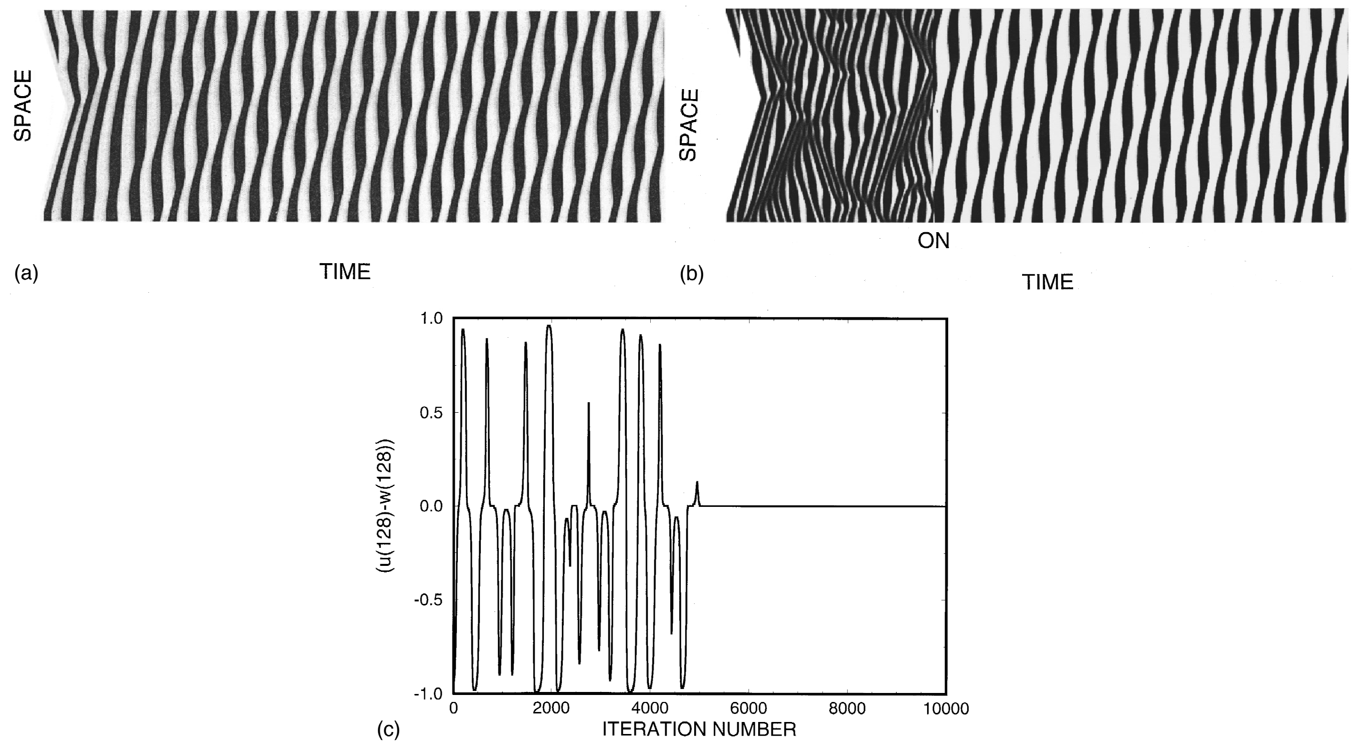

FIG. 3. Space-time plots for nonidentical one-dimensional chemical models [Eqs. (1) and (2)] studied under periodic boundary conditions with the dimensionless system length $L=100$ and the number of grid sites $N=256$. (a) Dynamics of the drive system for the system parameters $a=0.84$, $\epsilon=0.09$, and $b=0.07$ exhibiting phase turbulence. (b) Dynamics of the response system at the system parameters $a=0.84$, $\epsilon=0.39$, and $b=-0.045$ prior to initiation of the synchronizing feedback, exhibiting qualitatively different turbulence (amplitude). However, subsequent to switching on the feedack (indicated by ''ON'' on the plot) with $K = 0.1$, the dynamics of the response system converges onto the dynamics of the drive system, namely, phase turbulence. (c) Difference in the local dynamics [(u_i-w_i), proportional to the synchronizing feedback] at grid point 128. It indicates clearly the synchronization of the dynamics subsequent to which the feedback vanishes.

summary, the dimensionless length (100) of the extended chemical oscillator is divided into 256 grid elements for both the drive and the response system and the difference between the local values of the activator variable (u_i-w_i) between the drive and the response system (multiplied by a gain K) is superimposed onto the dynamical evolution of the response system.

Figures 1(a) and 1(b) show the space-time plots (activator values) of two uncorrelated identical systems (same parameter values) initialized at different initial conditions. The synchronization feedback is initiated at the indicated value (''ON'') and the response system [Fig. 1(b)] almost instantly locks onto the dynamics of the drive system. This is evident from Fig. 1(c), which plots the difference of activator values (u_i-w_i) at the 128th grid element (local dynamics). Beyond iteration number 5000, the difference between the local activator values goes to zero, indicating successful synchronization. This difference is also proportional to the synchronizing feedback at that grid element (128th) as a function of time. Upon successful synchronization the feedback goes to zero as the dynamics are identical. This is analogous to controlling chaos since the control signal vanishes subsequent to the stabilization of the target orbit.

More remarkable are the results when the two systems are maintained at unequal parameter values and exhibit different dynamical behaviors. In Figs. 2(a) and 2(b) the drive system exhibits traveling pulse dynamics [Fig. 2(a)], whereas the response system shows turbulence [Fig. 2(b) prior to ''ON'']. Subsequent to triggering K beyond a threshold $(K>0.05)$ the synchronizing feedback forces the response system to exhibit stable traveling pulse dynamics. The convergence in the activator (u,w) concentrations (and vanishing of the feedback) at the 128th grid element is indicated in Fig. 2(c). This is what is referred to as generalized synchronization in the literature. Finally, we synchronize two systems exhibiting qualitatively different types of turbulence [18]. Figures 3(a) and 3(b) show the dynamics of the drive and response system prior and subsequent to the initiation of the synchronizing feedback $(K>0.05)$. The convergence of the dynamics and vanishing of the feedback at the 128th grid element are shown in Fig. 3(c). It should be pointed out that the results remain unchanged even if one uses the inhibitor variables for the feedback.

In conclusion, using a continuous feedback synchronization (regular and generalized) of spatially extended systems is achieved. Possible applications of the synchronization of extended systems to various physical, chemical, and biological systems are discussed in detail in a recent article by Kocarev and Parlitz [19]. Our main motivation for this work was its possible relevance to biological systems. Using this technique, one has flexible control over the output dynamical behavior of the response system (from stable dynamics to phase turbulence), which can be varied according to the particular requirement of a situation. The potential drawback of our technique at this point is that it requires knowledge of local dynamics in order to implement the synchronization. In the future, we plan to investigate the possibility of attaining synchronization via global feedback techniques making it easier to implement in real systems.

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